59.

$$V = c(r_0 - r)r^2$$

$$V' = c\left[(r_0 - r)2r + r^2(-1)\right] = 0$$

$$2rr_0 - 2r^2 - r^2 = 0$$

$$2rr_0 - 3r^2 = 0$$

$$r(2r_0 - 3r) = 0$$

$$r = 0 \qquad 2r_0 - 3r = 0$$

$$r = \frac{2}{3}r_0$$

$$V'' = 2r_0 - 6r$$

$$V''\left(\frac{2}{3}r_0\right) = 2r_0 - 6 \cdot \frac{2}{3}r_0$$

$$= 2r_0 - 4r_0$$

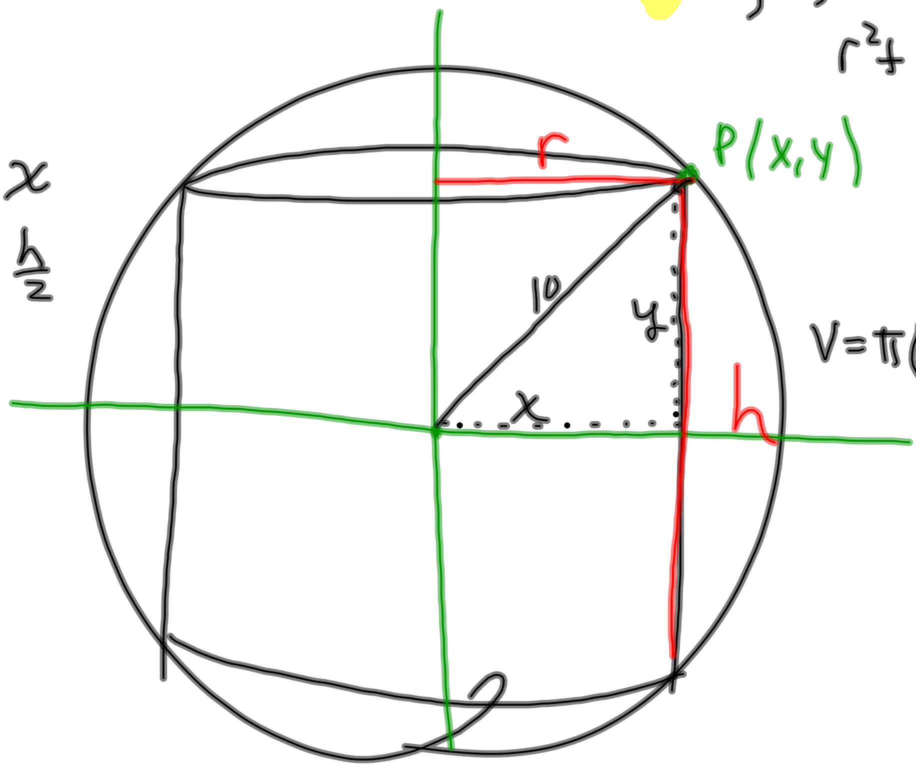
$$= -2r_0 < 0$$

Oct 23-7:39 AM

22

$$r = x$$

$$y = \frac{h}{2}$$



$$V = \pi r^2 h \; ; \quad x^2 + y^2 = 10^2$$

$$r^2 + \frac{h^2}{4} = 10^2$$

$$r^2 = 100 - \frac{h^2}{4}$$

$$V = \pi\left(100 - \frac{h^2}{4}\right) \cdot h$$

Oct 23-8:14 AM

4.4c Modeling and Optimization

Examples from Economics

Maximum Profit: If there is a maximum profit, it occurs when marginal revenue = marginal cost

P = profit R = revenue C = cost

max P: $P = R - C$

$P' = R' - C' = 0$

$R' = C'$

R & C might be functions of x

Oct 25-8:00 AM

Suppose r(x) = 9x and c(x) = x^3 - 6x^2 + 15x, where x represents 1000's of units. Is there a production level that maximizes profit? If so, what is it?

$P = 9x - (x^3 - 6x^2 + 15x)$

$9 = 3x^2 - 12x + 15$

$x = .586$

$x = 3.414$

x = 3.414 maximizes profit.

$P' = 9 - (3x^2 - 12x + 15) = 0$

$P'' = -6x + 12$

$P''(.586) > 0$

$P''(3.414) < 0$

Oct 25-8:05 AM

average cost = c(x)/x

x = production level
(how many you make)

Minimum Average Cost: If there is a minimum average cost, it occurs when average cost = marginal cost.

$$\frac{c(x)}{x} = c'(x)$$

minimize

$$y = \frac{c(x)}{x}$$

$$y' = \frac{x \cdot c'(x) - c(x) \cdot 1}{x^2} = 0$$

$$x \cdot c'(x) - c(x) = 0$$

$$x \cdot c'(x) = c(x)$$

$$c'(x) = \frac{c(x)}{x}$$

Oct 25-8:06 AM

Suppose c(x) = x^3 - 6x^2 +15x, where x represents 1000's of units. Is there a production level that minimizes average cost? If so, what is it?

$$y = \text{ave cost} = \frac{c(x)}{x} = \frac{x^3 - 6x^2 + 15x}{x} = x^2 - 6x + 15$$

$$y' = 2x - 6 = 0$$

$$x = 3$$

$$y'' = 2 > 0$$

$$x^2 - 6x + 15 = 3x^2 - 12x + 15$$

$$x = 3$$

$$\cancel{x = 0}$$

Oct 25-8:08 AM

Oct 23-7:24 AM